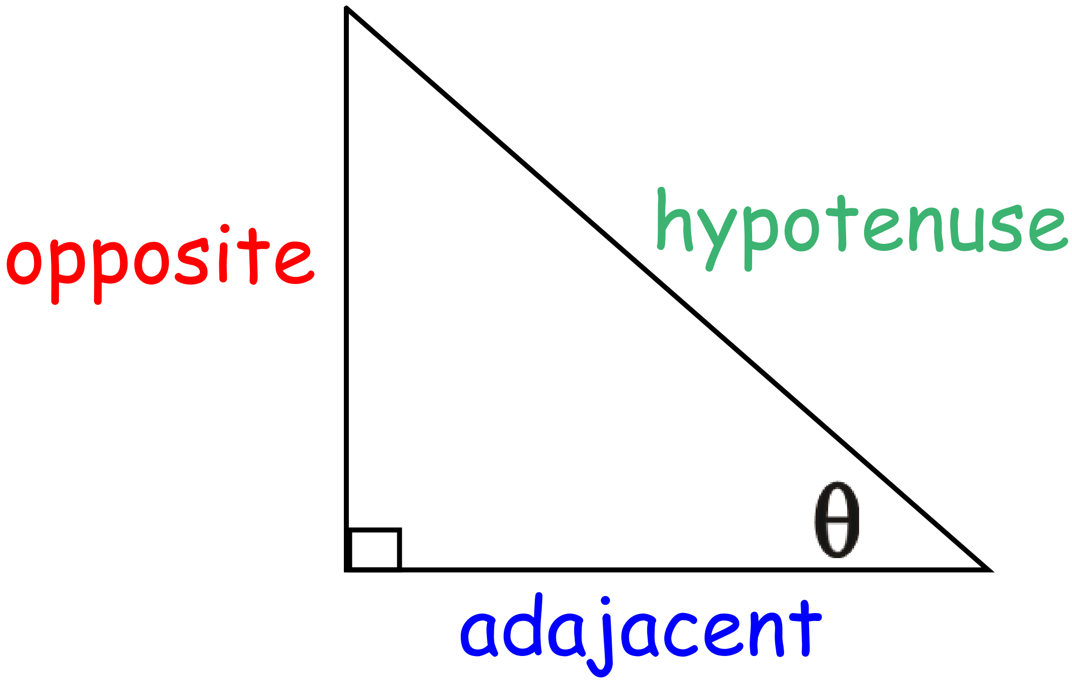
6.2 Right Triangle Trig

Seemed like winter would never end.



$\theta \quad \beta \quad \alpha \Rightarrow$ variable for angles

<u>Notes</u>

Right Triangle Trigonometry

The Six Trigonometric Functions

$\underbrace{\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp}}_{\text{always be less than 1}}$ $\qquad \underbrace{\tan\theta = \frac{opp}{adj}}_{\text{any thing}}$

$\underbrace{\csc\theta = \frac{hyp}{opp} \qquad \sec\theta = \frac{hyp}{adj}}_{\text{always be greater than 1}}$ $\qquad \underbrace{\cot\theta = \frac{adj}{opp}}_{\text{anything}}$

* some exceptions

Where opp = the length of the side *opposite* θ

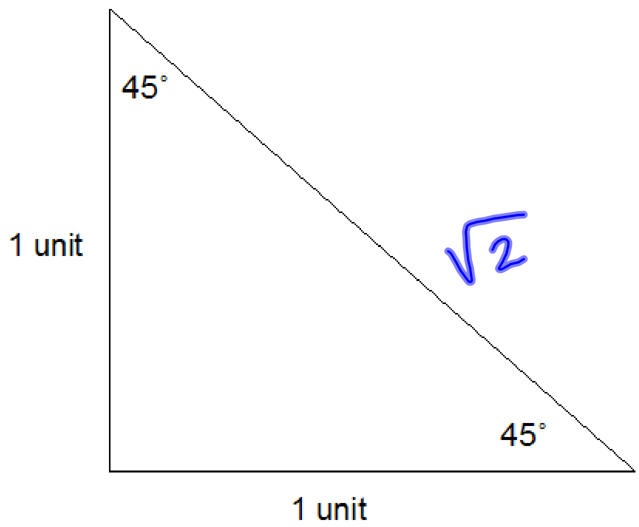
adj = the length of the side *adjacent to* θ

hyp = the length of the *hypotenuse*

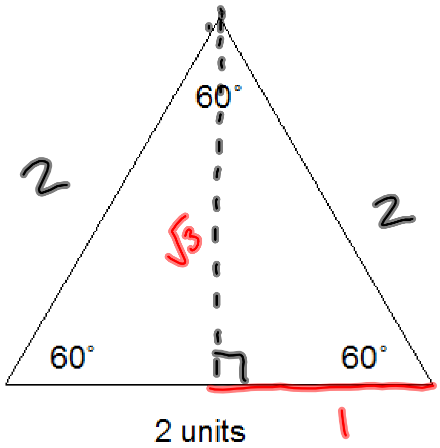
Domain: angles

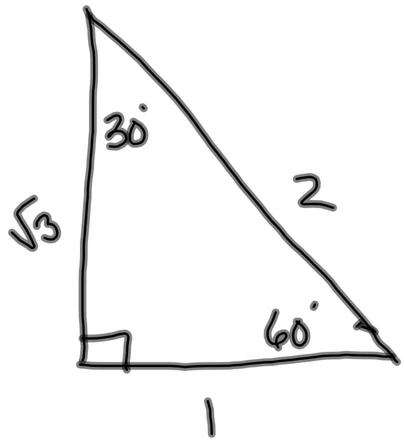
Range: ratios

45°-45°-90° Triangle



30˚-60˚-90˚ Triangle





Special Angles

sin 30° = $\frac{1}{2}$	cos 30° = $\frac{\sqrt{3}}{2}$	tan 30° = $\frac{\sqrt{3}}{3}$
sin 45° = $\frac{1}{\sqrt{2}}$ $\frac{\sqrt{2}}{2}$	cos 45 ° = $\frac{\sqrt{2}}{2}$	tan 45° = 1
sin 60° = $\frac{\sqrt{3}}{2}$	cos 60 ° = $\frac{1}{2}$	tan 60° = $\sqrt{3}$

Cofunction Relationships…

- Sin (90° - θ) = Cos θ
- Cos (90° - θ) = Sin θ
- Tan (90° - θ) = Cot θ
- Csc (90° - θ) = Sec θ
- Sec (90° - θ) = Csc θ
- Cot (90° - θ) = Tan θ

$\sin 82^\circ = \cos 8^\circ$

$\tan 26^\circ = \cot 64^\circ$

$\sec 34^\circ = \csc 56^\circ$

$\cancel{\cot 15^\circ = \sin 75^\circ}$

Trigonometric Identities

A. Reciprocal Identities:

$\sin\theta = \dfrac{1}{\csc\theta}$ $\qquad$ $\cos\theta = \dfrac{1}{\sec\theta}$ $\qquad$ $\tan\theta = \dfrac{1}{\cot\theta}$

NOT $\sin^{-1}$

$\csc\theta = \dfrac{1}{\sin\theta}$ $\qquad$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\qquad$ $\cot\theta = \dfrac{1}{\tan\theta}$

Special Reciprocal Angles

csc 30° = 2	sec 30° = $\frac{2\sqrt{3}}{3}$	cot 30° = $\sqrt{3}$
csc 45° = $\sqrt{2}$	sec 45 ° = $\sqrt{2}$	cot 45° = 1
csc 60° = $\frac{2\sqrt{3}}{3}$	sec 60 ° = 2	cot 60° = $\frac{\sqrt{3}}{3}$

Trigonometric Identities (cont.)

- Quotient Identities:

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$\sin 60° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$

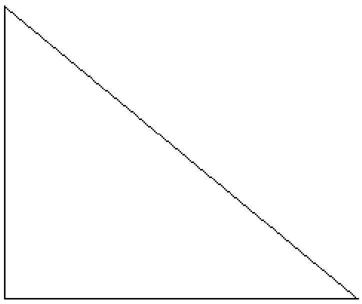
a) $\tan 60°$

b) $\sin 30°$

c) $\cos 30°$

Trig Identities (cont.)

- Pythagorean Identities:



$\sin^2\theta + \cos^2\theta = 1$ $\quad$ $\tan^2\theta + 1 = \sec^2\theta$ $\quad$ $1 + \cot^2\theta = \csc^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$1+\cot^2\theta=\csc^2\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}$$

$$\tan^2\theta+1=\sec^2\theta$$

Ex 1 If θ is an acute angle such that $\cos\theta = \frac{3}{10}$ then find the following:

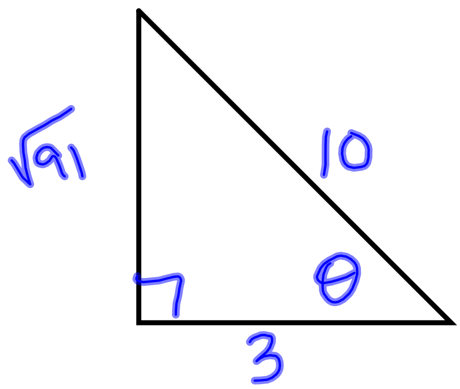
a) $\sin\theta$ $\frac{\sqrt{91}}{10}$

b) $\tan\theta$ $\frac{\sqrt{91}}{3}$

c) $\cot\theta$ $\frac{3\sqrt{91}}{91}$

d) $\sec\theta$ $\frac{10}{3}$

e) $\csc\theta$ $\frac{10\sqrt{91}}{91}$



Ex 2 Use trig identities to transform one side of the equation into the other.

a) $\csc\theta \tan\theta = \sec\theta$

$\frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta}$ =

$\frac{1}{\cos\theta}$ =

$\sec\theta = \sec\theta$

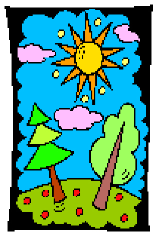
b) $\frac{\cot\theta + \tan\theta}{\cot\theta} = \sec^2\theta$

$\frac{\cot\theta}{\cot\theta} + \frac{\tan\theta}{\cot\theta}$ =

$1 + \tan^2\theta$ =

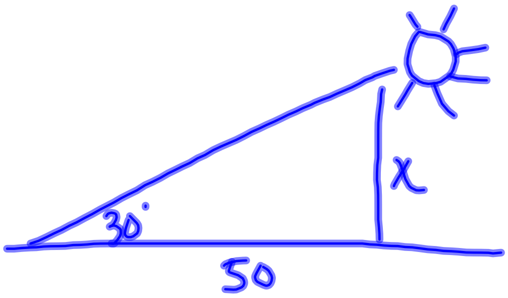
$\sec^2\theta = \sec^2\theta$

Applications



Ex 4

If the sun is 30° up from the horizon and shining on a tree forming a 50-foot shadow, how tall is the tree?



$$\sin 30^\circ = \frac{x}{50}$$

$$\frac{1}{2} = \frac{x}{50}$$

$$x = 25 \text{ ft}$$